

Child-Care Policies in Pay-As-You-Go Pension*

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Abstract

Some economically developed countries are suffering from an aging society with fewer children which is brought about the burden of social security. Child allowance and child-care service are provided by the government in these countries to raise the fertility. An increase in the fertility pulls up the labor population in future. An increase in labor population can provide enough social security benefit as pension and so on. This paper considers two child care policies: one for child allowances and the other for the subsidy for child-care service. These two policies can raise the fertility and labor population in future. This paper derives that child allowances and the subsidy for child-care service are substantially different each other in terms of the effects on pension benefit even if these policies are same about decrease in the child-care cost. Child allowances can not always raise the pension benefit because child allowances has the negative effect on labor supply. On the other hand, the subsidy for child-care service can always raise the pension benefit because stimulates labor supply.

Keywords: Aging Society, Child-Care Service, Fertility, Labor Supply, Pay-As-You-Go Pension

JEL Classifications: H51, H55, J14

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1 Introduction

Some economically developed countries are suffering from an aging society with fewer children which is brought about the burden of social security. Pension benefit can be provided by enough younger people and labor supply. In economically developed countries, the fertility and female labor supply are high level in Sweden and France. On the other hand, the fertility and female labor supply are low level in Japan. The government in Japan is providing child allowance and the subsidy for child-care service. However, it is said that the subsidy for child-care service or the quantity of child-care service is not enough. The reason of low female labor supply in Japan is attributed by not enough child-care service. These child-care policies should be provided to maintain the level of pension benefit in future.

[Insert Fig.1 around here.]

Fig.1 shows the social expenditure for child-care policies and the fertility. In France and Sweden, the fertility and the social expenditure for child-care policies is higher than that in Japan. Therefore, we think that the low fertility in Japan can be pulled up by the enough social expenditure for child-care policies. However, it is not good that only cash benefit is given for children. In France and Sweden, female labor participation is higher than that in Japan.¹ As shown by Sleenbos (2003), in OECD countries, female labor participation and fertility are positive correlation, then the government in Japan should provide the policy that parents can continue working with children.

Large fertility and large labor participation rate are needed to maintain the pension system because the government must collect the revenue for large amount of pension benefit in an aging society in OECD countries. Fig.2 shows the replacement rate of pension benefit OECD countries.

[Insert Fig.2 around here.]

¹The female employment rate from 25 years old to 54 years old in Japan 69.2% at 2012. This is smaller than that in France and Sweden, which are 76.0% in France and 82.5% in Sweden (Data: OECD Statistics).

If the replacement rate of pension benefit is large, the pension benefit for the income of working generation is large. The replacement rate in Japan is low. The pension system in Japan is reformed by the government at 2004. The reform maintains the replacement rate of pension benefit as 50%. It is clear that only increase in contribution rate of pension benefit can not maintain the 50% of replacement rate in future because the rate of older people for total population will increase and the working population will decrease. Therefore, the government should increase the fertility and the labor participation rate with policies to maintain the pension benefit level. This paper examines how the government should provide child-care policies to raise the pension benefit.

Many earlier studies examine the child-care policies. Backer (1960), Becker and Barro (1988) and Barro and Becker (1989) set an endogenous fertility model and examine how the fertility is determined. As the studies that examines child-care policies in an endogenous fertility model, Zhang (1997), van Groezen, Leers and Meijdam (2003), van Groezen and Meijdam (2008), Fanti and Gori (2009), Oshio and Yasuoka (2009) and Yasuoka and Goto (2011) set an endogenous fertility with child-care policies and examine how the child-care policies affects the fertility. van Groezen, Leers and Meijdam (2003) and van Groezen and Meijdam (2008) show the positive effect of child allowances on the fertility. However, Fanti and Gori (2009) shows the negative effect because of a decrease in household's income brought about by a decrease in capital stock per capita. Zhang (1997) shows that child allowances can raise the fertility, however, this policy reduces education investment for children, which is the substitution between quality and quantity of children. Oshio and Yasuoka (2009) shows that child allowances are needed to stop decreasing the fertility in pay-as-you-go pension.

Ahn and Mira (2002) and Sleetbos (2003) show that the relationship between the fertility and female labor participation changes from negative one to positive one. One of the reason is child-care service provided in the market. Galor and Weil (1996) derives negative relationship because an increase of female wage stimulates female labor participation and decreases child-care time and the fertility because of high opportunity cost. However, Apps and Rees (2004) and Martínez and Iza (2004) and Day (2012) consider child-care service and shows the positive

relationship between the fertility and the female labor supply. Momota (2000) considers the child-care policy affects the child-care time or labor supply. Yasuoka and Miyake (2010) shows that child-care service does not increase the fertility and decrease child-care time because the price of child-care service increases and reduce to use child-care service. Hirazawa and Yakita (2010) considers the model that the fertility is determined by child-care service and child-care time as Apps and Rees (2004). Hirazawa and Yakita (2010) examines whether tax cut to provide pension can bring about Pareto improving pension or not. Many earlier papers examine how the pension policy affects the fertility or labor supply in an endogenous fertility model. Wigger (1999), Lin and Tian (2003) and Fenge and Meier (2005) examines how the pension policy affects on the fertility and the labor supply.

Many earlier studies derive that child-care policies need to raise the fertility. However, no study examine how the government should provide child-care policies to raise the pension benefit in an endogenous fertility model determined by child-care service and household's child-care time.²

This paper considers the endogenous fertility in pay-as-you-go pension and examines whether child-care policies can raise the fertility, that is, the younger people in future, or not and then can raise the pension benefit or not thanks to an increase in labor supply given by an increase in younger people. If the pension benefit is provided by the revenue that is levied labor income at the contribution rate, the pension benefit depends on both fertility (the intergenerational population ratio) and aggregate labor supply. If the fertility is determined by both child-care service and child-care time by parents, child allowance and the subsidy for child-care service can raise the fertility and alleviate an aging society with fewer children because these policies brings about a decrease in child-care cost has incentives to have more children. However, these policies have the different effect on the pension benefit. This paper derives that child allowances and the subsidy for child-care service are substantially different each other in terms of the effects on pension benefit even if these policies are same about decrease in the child-care cost. Child allowances can not always raise the pension benefit because child allowances has the negative

²Hirazawa and Yakita (2010) derives that the cut for contribution rate can raise the pension benefit because of an increase in aggregate labor supply and aggregate revenue. However, no child-care policy is examined.

effect on labor supply. On the other hand, the subsidy for child-care service can always raise the pension benefit because the subsidy stimulates labor supply.

This paper consists of the followings. Section 2 sets the model and Section 3 derives the equilibrium, Section 4 examines whether child-care policies can raise the pension benefit or not and final section concludes this paper.

2 The Model

The model economy in this paper is constructed in terms of a two-period (young and old) overlapping generations model. The economy comprises agents of three types: households, two types of firm (one produces child care services and the other produces final goods) and a government. In t period, the population of younger people N_t and the population of older people N_{t-1} . We explain the agents in the following subsections.

2.1 Households

Individuals in households live in two period; young period and old period. Younger people provide labor supply to gain labor income. The labor income is allocated to consumption in younger period c_{1t} , the saving to consume in old period c_{2t+1} and purchasing child-care service e_t at the price p_t . The individuals allocate their time to labor time $(1 - l_t)$ and child-care time l_t . This paper assumes the quantity of children as follows.

$$n_t = Al_t^\epsilon e_t^{1-\epsilon}, \quad 0 < A, \quad 0 < \epsilon < 1. \quad (1)$$

Apps and Rees (2004) assumes the fertility function as constant return to scale about l_t and e_t . This paper specifies the constant return to scale function to this Cobb-Douglas function as assumed by Hirazawa and Yakita (2010).³ With wage rate as w_t , the individuals gain the labor income $(1 - l_t)w_t$. The government provides pension benefit for older people S_t and collects revenues from labor income brought about by younger people at contribution rate τ . In addition, the government provides child-care policies: one for child allowance and the other for subsidy

³Marínez and Iza (2004) assumes perfectly substituting function between l_t and e_t .

for child-care service. Then, the household's budget constraint is given by

$$c_{1t} + \frac{c_{2t+1}}{R_{t+1}} + (1 - \gamma)p_t e_t = (1 - \tau)(1 - l_t)w_t + \frac{S_{t+1}}{R_{t+1}} - T_t + q_t n_t, \quad (2)$$

where R_{t+1} denotes an interest rate of the annuity. The government provides the subsidy for child-care service at γ rate and child allowances at q_t . These policies is financed by T_t . The household's utility function is assumed by⁴

$$u = \alpha \ln n_t + \beta \ln c_{1t} + p(1 - \alpha - \beta) \ln c_{2t+1}, \quad 0 < \alpha, 0 < \beta, \alpha + \beta < 1. \quad (3)$$

p denotes the survival rate in old period ($0 < p < 1$). The $1 - p$ ratio of population in each generation can live only in younger people and can not live in old period. If the younger people die and can not live in old period, their annuity is distributed for the older people who live in old period. The households decide the optimal allocation to maximize their utility (3) subject to the function of quality of children (1) and the budget constraint (2) as follows,

$$c_{1t} = \frac{\beta}{\alpha + \beta + p(1 - \alpha - \beta)} \left((1 - \tau)w_t - T_t + \frac{S_{t+1}}{R_{t+1}} \right), \quad (4)$$

$$c_{2t+1} = \frac{p(1 - \alpha - \beta)}{\alpha + \beta + p(1 - \alpha - \beta)} \left((1 - \tau)w_t - T_t + \frac{S_{t+1}}{R_{t+1}} \right), \quad (5)$$

$$l_t = \frac{\alpha \epsilon}{\alpha + \beta + p(1 - \alpha - \beta)} \frac{1}{(1 - \tau)w_t} \left((1 - \tau)w_t - T_t + \frac{S_{t+1}}{R_{t+1}} \right) + \frac{\epsilon q_t n_t}{(1 - \tau)w_t}, \quad (6)$$

$$e_t = \frac{\alpha(1 - \epsilon)}{\alpha + \beta + p(1 - \alpha - \beta)} \frac{1}{(1 - \gamma)p_t} \left((1 - \tau)w_t - T_t + \frac{S_{t+1}}{R_{t+1}} \right) + \frac{(1 - \epsilon)q_t n_t}{(1 - \gamma)p_t}, \quad (7)$$

As shown by (6) and (7), and increase in child allowance q_t raises both l_t and e_t . On the other hand, an increase in the subsidy for child-care service γ raises only e_t . These policies has the positive effect to raise the fertility. An increase in p , which brings about the aging effect, decreases both l_t and e_t and then decreases fertility n_t as shown by many earlier papers.

2.2 Firms

In this model, there are two sectors: one for final goods sector and the other for elderly care sector. The production function in the final goods sector is assumed as follows,

$$Y_t = F(K_t, B_t L_t), \quad \frac{\partial Y_t}{\partial K_t} > 0, \quad \frac{\partial Y_t}{\partial B_t L_t} > 0, \quad \frac{\partial^2 Y_t}{\partial K_t^2} < 0, \quad \frac{\partial^2 Y_t}{\partial (B_t L_t)^2} < 0, \quad \frac{\partial Y_t}{\partial K_t \partial B_t L_t} > 0. \quad (8)$$

⁴Yakita (2001) and van Groezen and Meijdam (2008) consider the uncertain lifetime in old period and examines the relationship of the fertility.

where K_t and L_t denote capital stock and effective labor. B_t shows labor productivity. Defining $\frac{Y_t}{B_t L_t} \equiv f'(k_t)$ and $k_t \equiv \frac{K_t}{B_t L_t}$ and assuming competitive market and small open economy, an interest rate $1 + r_t = f'(k_t)$ is fixed by world interest r and k is fixed. The capital stock is fully depreciated in a period. Then, the wage rate is given by $w_t = B_t w$, where $w = f(k_t) - f'(k_t)k_t$. In addition, considering $R_t = \frac{1+r_t}{\rho}$, R_t is fixed by constant R .

Next, we consider the child-care sector. The child-care service is produced by the following product function.

$$Y_t^c = \rho L_t^c, \rho > 0. \quad (9)$$

The child-care service is produced by only labor input. This function is assumed by Hashimoto and Tabata (2010) and Yasuoka and Miyake (2010).⁵ In putting labor demand for child-care service L_t^c into the child-care sector, the profit π_t is given by,

$$\pi_t = p_t \rho L_t^c - w_t^c L_t^c. \quad (10)$$

Aggregate supply of elderly care service is $X_t = \rho L_t^c$. The wage rate w_t^c is given by

$$w_t^c = \rho p_t. \quad (11)$$

Considering homogenous household and complete labor mobility, the wage in child-care sector w_t^c is given by

$$w_t^c = B_t w \quad (12)$$

or

$$p_t = \frac{B_t w}{\rho}. \quad (13)$$

If the B_t grows at the constant rate g , we obtain $w_{t+1} = (1 + g)w_t$ and we find the price of child-care service p_t increases with B_t .

2.3 Government

The government in this model economy provides two child care policies and pay-as-you-go pension. First, the government levies taxation on the younger people to provide child allowance and

⁵Yasuoka and Miyake (2010) assumes the same function form as child care service sector.

to subsidize elderly care service. Considering balanced budget reduces to the following equation,

$$T_t = q_t n_t + \gamma p_t e_t, \quad (14)$$

where $T_t = \theta w_t$, $q_t = \hat{q} w_t$, $0 < \theta < 1$, $0 < \hat{q} < 1$, $0 < \gamma < 1$.

Second, the government provides pay-as-you-go pension that the government collects the revenue from the younger people at t period and gives the benefit for the older people in same t period. Considering balanced budget and $n_{t-1} = \frac{N_t}{N_{t-1}}$, pension benefit S_t is given by $S_t = \frac{n_{t-1} \tau (1-l_t) w_t}{p}$, that is,

$$x_t = \frac{n_{t-1} \tau (1-l_t)}{p}, \quad (15)$$

where $\frac{S_t}{w_t} = x_t$. We can consider x_t as the replacement rate of pension benefit.

3 Equilibrium

The child-care time l_t and purchase child-care service e_t are given by the followings.

$$l_t = \frac{\epsilon}{1-\tau} \left(C \left(1 - \tau - \theta + \frac{1+g}{R} x_{t+1} \right) + \hat{q} n_t \right), \quad (16)$$

$$e_t = \frac{\rho(1-\epsilon)}{1-\gamma} \left(C \left(1 - \tau - \theta + \frac{1+g}{R} x_{t+1} \right) + \hat{q} n_t \right), \quad (17)$$

where $C = \frac{\alpha \epsilon}{\alpha + \beta + p(1-\alpha-\beta)}$. Then, the fertility n_t is given by

$$n_t = \frac{A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon \left(\frac{\rho(1-\epsilon)}{1-\gamma} \right)^{1-\epsilon} C \left(1 - \tau - \theta + \frac{1+g}{R} x_{t+1} \right)}{1 - \hat{q} A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon \left(\frac{\rho(1-\epsilon)}{1-\gamma} \right)^{1-\epsilon}}. \quad (18)$$

Considering that n_{t-1} and l_t depend on x_t and x_{t+1} as given by (18) and (16), respectively, (15) shows the dynamic equation of x_t . This paper assumes small open economy and the equilibrium is obtain if x_t is given. The locally stable condition is derived as the following inequality,⁶

$$-1 < \frac{1 - A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \frac{C\tau(1+g)}{\rho R} (1-l)}{-\frac{n\tau\epsilon C(1+g)}{\rho R(1-\tau)}} < 1. \quad (19)$$

⁶See Appendix for a detailed proof.

4 Policy Effect

In economically developed countries, child-care policies are provided to raise the fertility and then raise the labor supply in future. This section examines whether child-care policies can raise the pension benefit or not. This section considers two child-care policies: one for child allowances which give the allowance in proportion to the quantity of children and the other for the subsidy for child-care service. First of all, this paper examines the effects of child allowances.

4.1 Child allowance

Considering (14)-(16), (18), $\gamma = 0$ and differentiating x by θ at the approximation of $\hat{q} = 0$, the sign of $\frac{dx}{d\theta}$ is ambiguous as shown by

$$\frac{dx}{d\theta} = \frac{\frac{\tau(1-C)}{\rho} \left(A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} (1-l) - \frac{\epsilon n}{1-\tau} \right)}{1 + \frac{\tau C(1+g)}{\rho R} \left(\frac{\epsilon n}{1-\tau} - (1-l) A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \right)}. \quad (20)$$

Considering the locally stable steady state, the denominator of (20) is positive. Therefore, if the numerator of (20) is positive, the sign of (20) is positive, that is, child allowances can raise the pension benefit x . The inequality condition to be positive sign is

$$x < \frac{1 - 2\epsilon C}{\frac{2\epsilon C(1+g)}{R(1-\tau)}}. \quad (21)$$

However, x is non-negative valuables. With $1 - 2\epsilon C < 0$, that is, $\epsilon > \frac{\alpha+\beta+p(1-\alpha-\beta)}{2\alpha}$, it exists no x to hold the inequality condition (21). Therefore, child allowances can not raise the pension benefit x . Then, the following proposition is established.

Proposition 1 With $x < \frac{1-2\epsilon C}{\frac{2\epsilon C(1+g)}{R(1-\tau)}}$, child allowances can raise the pension benefit x in the steady state. However, if $\epsilon > \frac{\alpha+\beta+p(1-\alpha-\beta)}{2\alpha}$, child allowances can not raise the pension benefit x though (21) is held.

The first term in bracket of (20) in numerator shows that an increase in the fertility raises the pension benefit x directly. $\frac{dn}{d\theta}$ is derived as

$$\frac{dn}{d\theta} = A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \left(1 - C + \frac{C(1+g)}{R} \frac{dx}{d\theta} \right). \quad (22)$$

The first term in right hand side shows that child allowances can directly raise the fertility $\frac{\partial n}{\partial \theta}$. However, this paper sets the pension benefit financed by labor income. Therefore, the change of labor supply time $1 - l$ affects the pension benefit x . This effect shows the second term in bracket of (20) in numerator, which is negative sign. An increase in child allowances raises directly child-care time l and decreases labor supply time. $\frac{dl}{d\theta}$ is derived as

$$\frac{dl}{d\theta} = \frac{\epsilon}{1 - \tau} \left(1 - C + C \frac{1 + g}{R} \frac{dx}{d\theta} \right). \quad (23)$$

The term $1 - C$ shows that child allowances can directly raise the child-care time. Therefore, if this effect is large, the pension benefit decreases even if the fertility (the younger population in future) increases. The pension benefit x in providing child allowances decides whether child allowances can raise the pension benefit or not, that is, whether the effect of an increase in the fertility is larger than the one of a decrease in labor supply time or not. we obtain $\frac{dx}{d\theta} = \frac{\tau}{\rho} \left((1 - l) \frac{dn}{d\theta} - n \frac{dl}{d\theta} \right)$ with (15). Small pension benefit x means small fertility and weakens the effect of $\frac{dl}{d\theta}$. However, if $\epsilon < \frac{\alpha + \beta + p(1 - \alpha - \beta)}{2\epsilon}$, the effect that labor supply time is decreased by child allowances is large, child allowances can not be used as the means to pull up the pension benefit though (21) is held.

Moreover, this paper explains child allowances policy in terms of economic growth and an aging society. As shown by (21), an increase in $1 + g$ decreases the rage of x which holds (21) because an increase in $1 + g$ enlarges the decrease in labor supply time. An increase in the older people increases, which means an increase in p , the range of ϵ which holds $\epsilon > \frac{\alpha + \beta + p(1 - \alpha - \beta)}{2\alpha}$ becomes small. That is, an increase in p reduces the fertility n as shown by (18) and the effects of child allowances is weaken. Then, child allowances can not raise the pension benefit.

4.2 Subsidy for Child-Care Service

Now, this subsection examines whether the subsidy for child-care service can raise the pension benefit or not. Cosidering (14)-(16), (18), $\hat{q} = 0$ and differntiating x by θ at the approximation of $\gamma = 0$, the sign of $\frac{dx}{d\theta}$ is positive as shown by

$$\frac{dx}{d\theta} = \frac{\frac{\tau \epsilon C n}{\rho(1 - \tau)}}{1 + \frac{\tau C(1 + g)}{\rho R} \left(\frac{\epsilon n}{1 - \tau} - (1 - l) A \left(\frac{\epsilon}{1 - \tau} \right)^\epsilon (\rho(1 - \epsilon))^{1 - \epsilon} \right)} > 0. \quad (24)$$

The subsidy for child-care service can always raise the pension benefit. Then, the following proposition is established.

Proposition 2 The subsidy for child-care service can always the pension benefit.

The effect of the subsidy for child-care service on the pension benefit are different from that of child allowances. The effect of child allowances on the pension benefit depends on pension benefit in providing the policy, economic growth rate and survival rate in old period. First, the subsidy for child-care service affects on the fertility as shown by

$$\frac{dn}{d\theta} = A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} C \frac{1+g}{R} \frac{dx}{d\theta}. \quad (25)$$

An increase in the subsidy has no direct positive effects on the fertility because an increase in the subsidy and tax burden are offset each other. Second, the subsidy for child-care service affects on the child-care time as shown by

$$\frac{dl}{d\theta} = \frac{\epsilon C}{1-\tau} \left(\frac{1+g}{R} \frac{dx}{d\theta} - 1 \right). \quad (26)$$

This is different from the case of child allowances. Child allowances decreases directly labor supply time (increases child-care time), which decreases the revenue for pension benefit. However, with the subsidy for child-care service, labor supply increases directly and then this effect increases the revenue to provide pension benefit.

5 Conclusions

This paper considers the endogenous fertility in pay-as-you-go pension and examines whether child-care policies can raise the fertility, that is, the younger people in future and can raise the pension benefit or not. If the pension benefit is provided by the revenue that is levied labor income at the contribution rate, the pension benefit depends on not only the fertility (the intergenerational population ratio) but also labor supply. If the fertility is determined by both child-care service and child-care time by parents, child allowances and the subsidy for child-care service can raise the fertility and alleviate an aging society with fewer children. This paper

derives that child allowances and the subsidy for child-care service are different each other in terms of the effect on the pension benefit.

This paper derives that child allowances and the subsidy for child-care service is substantially different each other in terms of the effects on pension benefit even if these policies is same in decrease in the child-care cost. Child allowances can not always raise the pension benefit because child allowances has the negative effect on labor supply. On the other hand, the subsidy for child-care service can always raise the pension benefit because stimulates labor supply.

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Appendix

The Stability Condition

Considering (15), (16), (18) and no child-care policy and differentiating at the steady state which holds $x_{t+1} = x_t = x$, we obtain

$$dx_t = \frac{\tau(1-l)}{\rho} dn_t - \frac{\tau n}{\rho} dl_t, \quad (27)$$

$$dl_t = \frac{\epsilon C(1+g)}{(1-\tau)R} dx_{t+1}, \quad (28)$$

$$dn_t = \frac{AC(1+g)}{R} \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} dx_{t+1}. \quad (29)$$

These equation reduces to $\frac{dx_{t+1}}{dx_t}$ as

$$\frac{dx_{t+1}}{dx_t} = \frac{1 - A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \frac{C\tau(1+g)}{\rho R} (1-l)}{-\frac{n\tau\epsilon C(1+g)}{\rho R(1-\tau)}}. \quad (30)$$

The sign of $\frac{dx_{t+1}}{dx_t}$ is positive or negative. First, we consider the positive sign of $\frac{dx_{t+1}}{dx_t}$. Then, the locally stable condition $0 < \frac{dx_{t+1}}{dx_t} < 1$ reduces to

$$0 < 1 + \frac{\tau C(1+g)}{\rho R} \left(\frac{\epsilon n}{1-\tau} - (1-l)A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \right) < \frac{\epsilon C\tau n(1+g)}{\rho R(1-\tau)}. \quad (31)$$

Second, we consider the negative sign of $\frac{dx_{t+1}}{dx_t}$. Then, the locally stable condition $-1 < \frac{dx_{t+1}}{dx_t} < 0$ reduces to

$$0 < 1 - A \left(\frac{\epsilon}{1-\tau} \right)^\epsilon (\rho(1-\epsilon))^{1-\epsilon} \frac{C\tau(1+g)}{\rho R} (1-l) < \frac{\tau\epsilon Cn(1+g)}{\rho R(1-\tau)}. \quad (32)$$

Even if the sign of $\frac{dx_{t+1}}{dx_t}$ is both positive and negative, the denominator of $\frac{dx}{d\theta}$ is positive in the locally stable steady state.

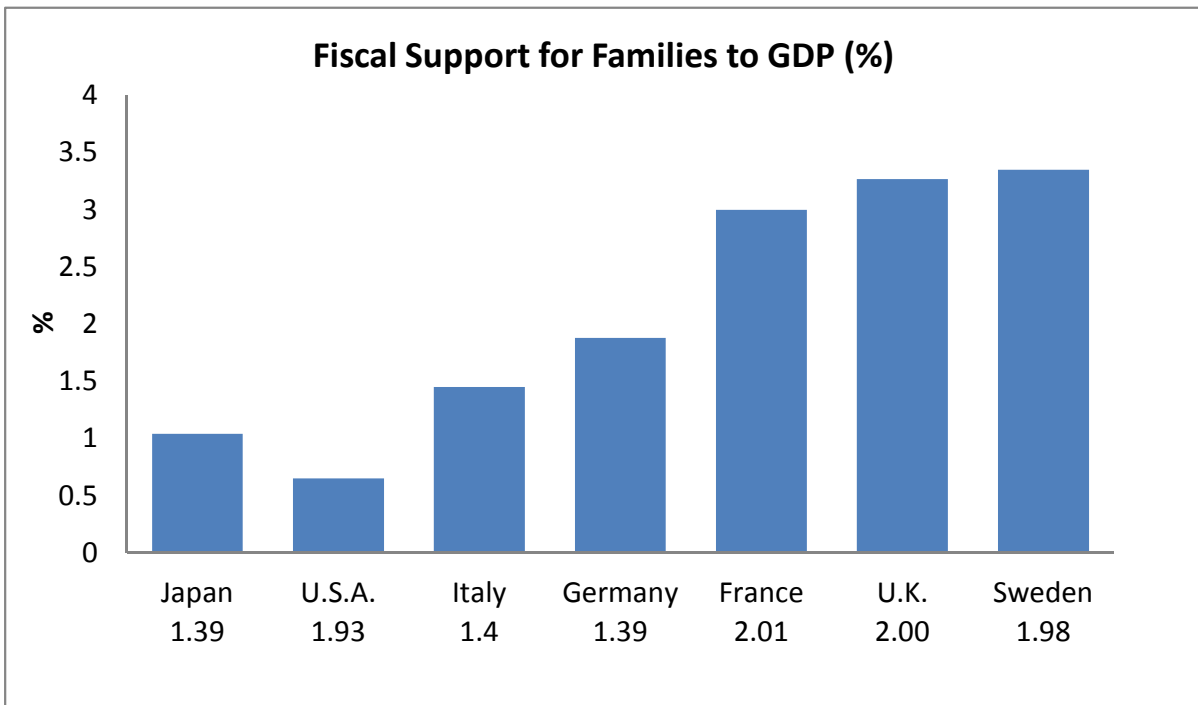


Fig.1: Fertility (below the country) and Fiscal Support for Family (share of Gross Domestic Product) (Data: OECD Social Expenditure Database (November 2008), A 2012 Declining Birthrate White Paper (2012), Demographic Yearbook (UN) and Vital Statistics in Japan (Ministry of Health, Labour and Welfare (in Japan).) Data of Fiscal Support for Families are those of 2007. Fiscal Support for Family includes benefits in kind (day-care/home help and other benefits in kind) and cash benefits (family allowance, maternity and parental leave and other cash benefit). Data of the total Fertility Rate are those of 2010.)

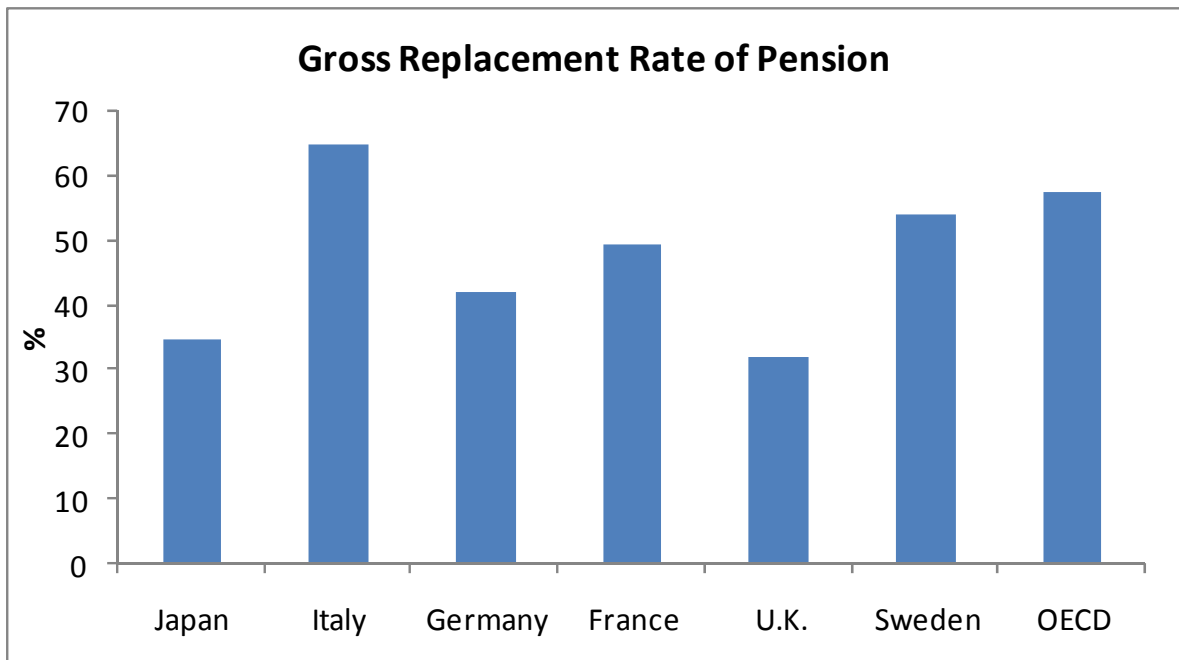


Fig. 2: Gross Replacement Rate of Pension (Data: OECD Statistic Pension at a Glance 2011. Gross Replacement Rate of Pension in Fig.1 shows the pension that men earn average wage income.)